Kinetic Equations

I. Information-Theoretical Derivation of a General Equation of Motion for the One-Particle Distribution Function

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Mit Hilfe informationstheoretischer Argumente wird der Anfangszustand zur Zeit t_0 bei der Ableitung kinetischer Gleichungen eliminiert, indem man ihn auf Grund der Jaynesschen Methode aus der vorausgesetzten Kenntnis des Zustandes zur Zeit t bestimmt. Damit kann eine abgeschlossene allgemeine Bewegungsgleichung der Form $i\dot{n}_{1,t} = \boldsymbol{T} n_{1,t}$ erhalten werden.

1. Introduction

In order to derive kinetic equations one starts from the Liouville-equation, which can be transformed into the BBGKY-hierarchy by reduction. Breaking off the hierarchy [1–3] or evaluation of the hierarchy by Bogoljubov's synchronization assumption [4] then leads to closed, explicitly time-independent equations of motion for the one-particle distribution function, which are called kinetic equations (Vlasov-, Boltzmann-, Fokker-Planck-, Lenard-Balescu-equation and so on). However, each of these equations demands particular physical assumptions and suppositions, which allow for systematic corrections only under certain conditions. Only recently Pompe [5] succeeded in deriving from a uniform formalism the above-mentioned equations and the Rice-Allnat-equation by the aid of a rigorous separation of the perturbation expansion from the evolution of the hierarchy. More rigorously, this program will be performed in the present series of papers by using the functional formalism of classical statistics [6–8].

The functional evolution operator $\exp(-i\tau \boldsymbol{L})$ following from the Liouville-equation can be represented by $\exp(-i\tau \boldsymbol{L}^{\circ})\, \boldsymbol{S}_\tau$.

An exponential expansion for the $\boldsymbol{S}$-operator

$$\boldsymbol{S}_\tau = N_{\eta,\, \delta/\delta\eta} \exp(\boldsymbol{\Gamma}_\tau), \quad \tau \equiv t - t_0 \tag{1}$$

is obtained by a general linked-cluster theorem [6, 9]. The equations for reduced distribution functions gained by reduction of the statistical functional mainly depend on the linked-cluster sum $\boldsymbol{\Gamma}_\tau$, which thus becomes the central quantity of the method developed here.

The equations of motion for the reduced distribution functions obtained in compact formulation by means of the $\boldsymbol{S}$-operator harmonize with the equations derived by Cohen [2]. They still contain explicitly the time interval τ between the initial time t_0 and the observation time t and also the initial state itself. However, kinetic equations as defined above do not contain the interval τ and the initial state; they are closed equations for the motions of reduced distribution functions. The aim of this paper is to eliminate the initial state.

The evolution equation

$$n_{1,t} = \boldsymbol{G}_\tau\, n_{1,t_0} \tag{2}$$

allows to calculate the distribution $n_{1,t}$ from n_{1,t_0} by the evolution operator $\boldsymbol{G}_\tau$, which is generally nonlinear. For the elimination of the initial state Cohen [2] obtained n_{1,t_0} from (2) by iteration relative to powers of the density. This result is introduced into the equation of motion, which follows from (2) by time-differentiation

$$\dot{n}_{1,t} = \dot{\boldsymbol{G}}_\tau\, n_{1,t_0}. \tag{3}$$

Then he obtained formally

$$\dot{n}_{1,t} = \dot{\boldsymbol{G}}_\tau\, \boldsymbol{G}_\tau^{-1}\, n_{1,t}. \tag{4}$$

[1] S. Ono, J. Phys. Soc. Jap. **19**, 815 [1964].
[2] E. G. D. Cohen, Physica **28**, 1025 [1962].
[3] S. T. Choh and G. E. Uhlenbeck, Thesis, University of Michigan 1958.
[4] N. N. Bogoljubov, J. Phys. (USSR) **10**, 256, 265 [1946], english translation in "Studies in Statistical Mechanics", Vol. I, Amsterdam 1961.
[5] W. Pompe, Ann. Phys. Leipzig **20**, 326 [1968].
[6] P. Quaas, K. Voss, and P. Ziesche, Acta Phys. Hung. **24**, No. 1 [1968].
[7] U. Bahr, P. Quaas, and K. Voss, Acta Phys. Hung. **24**, No. 2 [1968].
[8] P. Quaas, K. Voss, and P. Ziesche, Acta Phys. Hung. **24**, No. 2 [1968].
[9] P. Ziesche, Commun. Math. Phys. **5**, 301 [1967].

This procedure has essential disadvantages. On the one hand, the convergence of the iteration cannot be proved in a rigorous mathematical sense. It cannot be proved that $\boldsymbol{G}_\tau^{-1} n_{1,t}$ ever is a positive definite distribution function. On the other hand, because of the complicated structure of Eq. (2) COHEN was not able to specify the general term of the iterative solution and with it the general structure of (4). Therefore a general partial summation for a systematic correction of known kinetic equations is difficult. Further difficulties seemed to appear in connection with so-called secular terms [10].

These disadvantages may be overcome by an inversion of Eq. (2) with the aid of information-theoretical arguments. An exact discussion shows that this equation is the answer given by information-theory to the question what may be said about $n_{1,t}$, if only n_{1,t_0} is known. Accordingly the initial state n_{1,t_0} may be expressed by the equation

$$n_{1,t_0} = \boldsymbol{G}_{-\tau}\, n_{1,t}\,, \tag{5}$$

if only $n_{1,t}$ is known. Thus n_{1,t_0} can be eliminated from (3):

$$\dot{n}_{1,t} = \dot{\boldsymbol{G}}_\tau \boldsymbol{G}_{-\tau}\, n_{1,t}\,. \tag{6}$$

There is no doubt that in this procedure the left-hand side of (5) is again a positively definite distribution function. Moreover, the general structure of $\boldsymbol{G}_{-\tau}$ can be given like that of $\boldsymbol{G}_\tau$. In this way partial summations can be performed comparatively easily. They lead to kinetic equations in arbitrary orders of the expansion relative to interaction and density [11].

In part 2 we investigated what may generally be said about the time evolution of macroscopic observables, i. e. about quantities which in contrast to the microscopic state of the system do not contain macroscopically irrelevant information. Similar to ANDREWS [12] and POMPE and VOSS [13] a formal equation of motion may be given which does not contain the initial state. With the results of the functional formalism of part 3 and 4 the general equation of motion of the one-particle distribution functions follows in part 5 from this formal equation of motion.

[10] J. R. DORFMAN and E. G. D. COHEN, J. Math. Phys. **8**, 282 [1967].

[11] U. BAHR, P. QUAAS, and K. VOSS, Z. Naturforsch. **23 a**, 638 [1968]; hereafter referred to as II.

2. Time Evolution of Macroscopic Observables

In the following, a macroscopic observable is thought to be every quantity A, which by an equation

$$A_t \equiv \int \frac{\mathrm{d}1 \ldots \mathrm{d}N}{N!}\, a_{1\ldots N}\, f_{1\ldots N,t} \tag{1}$$

is an average with the symmetrized distribution function $f_{1\ldots N,t}$ of the particle coordinates and momenta $i \equiv \{\mathfrak{r}_i, \mathfrak{p}_i\}$. The reduced distribution functions in particular may be treated as macroscopic observables. The time behaviour of a macroscopic observable is generally given with $f_{1\ldots N,t}$ by the Liouville-equation

$$i\frac{\partial}{\partial t} f_{1\ldots N,t} = \boldsymbol{l}_{1\ldots N} f_{1\ldots N,t}\,;$$

$$\boldsymbol{l}_{1\ldots N} = \sum_k^{1\ldots N} \boldsymbol{l}_k + \tfrac{1}{2}\sum_{k\neq l} \boldsymbol{l}_{kl}\,, \tag{2}$$

$$\boldsymbol{l}_k \equiv -i\frac{\mathfrak{p}_k}{m}\frac{\partial}{\partial \mathfrak{r}_k}, \qquad \boldsymbol{l}_{kl} \equiv i\frac{\partial u(|\mathfrak{r}_k - \mathfrak{r}_l|)}{\partial \mathfrak{r}_k}\left(\frac{\partial}{\partial \mathfrak{p}_k} - \frac{\partial}{\partial \mathfrak{p}_l}\right).$$

However, if it is possible at least by approximation to represent the time evolution of A_t by a closed, explicitly time-independent equation

$$i\frac{\partial}{\partial t} A_t = \boldsymbol{T} A_t\,, \tag{3}$$

then (3) is to be the kinetic equation for A_t. This definition is a consequent generalization of BOGOLJUBOV's conception of a kinetic equation [4]. BOGOLJUBOV demands for instance for the one-particle distribution function f_1 (in systems with a defined particle number N) that their time-dependence from the BBGKY-hierarchy be represented by the functional dependence $f_{12,t} = f_{12}[f_{i,t}]$ in the form of a closed kinetic equation.

The operator $\boldsymbol{T}$ is time-independent in general. How kinetic equations with time-independent $\boldsymbol{T}$ can be obtained from the general equation of motion derived in this paper will be shown in the following paper II. In the functional formalism of classical statistics [6–8] the macroscopic quantity A can be obtained by an equation

$$A_t = P\boldsymbol{A} F_t^{(N)}, \quad F_t^{(N)} = \int \frac{\mathrm{d}1 \ldots \mathrm{d}N}{N!}\, \eta_1 \ldots \eta_N f_{1\ldots N,t} \tag{4}$$

[12] F. C. ANDREWS, Proc. Nat. Acad. Sci. US **54**, 13 [1965].

[13] W. POMPE and K. VOSS, Ann. Phys. Leipzig **19**, 253 [1967].

corresponding to (1). Here η_k is a one-particle test-function depending on $k \equiv \{\mathfrak{r}_k, \mathfrak{p}_k\}$, $F_t^{(N)}$ is the generating statistical functional, $\boldsymbol{A}$ is the functional operator corresponding to the physical quantity $a_{1\ldots N}$, and P symbolizes the prescription to set all test-functions η_k equal to one. The Liouville-equation (2) appears in the functional formalism in the form

$$i\frac{\mathrm{d}}{\mathrm{d}t}F_t^{(N)} = \boldsymbol{L}F_t^{(N)}, \qquad F_t^{(N)} = \exp\{-i\tau\boldsymbol{L}\}F_{t_0}^{(N)}, \tag{5}$$

$$\boldsymbol{L} = \boldsymbol{L}^0 + \boldsymbol{L}^{\mathrm{v}} = \int \mathrm{d}1\ \eta_1 \boldsymbol{l}_1 \frac{\delta}{\delta\eta_1} + \int \frac{\mathrm{d}1\,\mathrm{d}2}{2!}\eta_1\eta_2\boldsymbol{l}_{12}\frac{\delta}{\delta\eta_1}\frac{\delta}{\delta\eta_2}.$$

However, it is impossible to derive an equation like (3) exactly from (5) and (4), because after time-differentiation one gets from (4) only

$$i\frac{\partial}{\partial t}A_t = P\boldsymbol{A}\boldsymbol{L}F_t^{(N)}. \tag{6}$$

Approximations or assumptions are necessary in order to derive a closed equation of motion for macroscopic observables. The most important method for the solution of this problem [12, 13] is to construct the phase-space distribution and thus the functional $F_{t_0}^{(N)}$ by means of information theory [14], if only A_{t_0} is known. Then the temporal behaviour of the statistical functional is given by the solution (5) of the Liouville-equation. Finally A_t can be determined with (4):

$$A_t = P\boldsymbol{A}\exp\{-i\tau\boldsymbol{L}\}KA_{t_0}. \tag{7}$$

The time-differentiation of (7) yields

$$i\frac{\partial}{\partial t}A_t = P\boldsymbol{A}\boldsymbol{L}\exp\{-i\tau\boldsymbol{L}\}KA_{t_0}, \tag{8}$$

taking into account that P, $\boldsymbol{A}$, $\boldsymbol{L}$ and the construction operator K are time-independent. At this point, a general equation of motion such as (3) can be given, if A_{t_0} in (8) can be expressed by A_t. For this purpose COHEN [2] obtains A_{t_0} by an inversion of the nonlinear equation (7) and eliminates it in (8). Thus the difficulties appear which have been mentioned in the introduction.

In order to overcome these difficulties one solves the problem posed in (3) in two steps by means of information-theory [15]. First the temporal derivative $\dot{A}_t$ is represented by (8) from the knowledge of A_{t_0}. Then A_{t_0} is determined, if A_t is known. The corresponding equation is analogous to (7):

$$A_{t_0} = P\boldsymbol{A}\exp\{i\tau\boldsymbol{L}\}KA_t. \tag{9}$$

Putting (9) into (8) one gets the general equation of motion for the macroscopic quantity A derived by means of information-theory

$$i\frac{\partial}{\partial t}A_t \tag{10}$$
$$= P\boldsymbol{A}\boldsymbol{L}\exp\{-i\tau\boldsymbol{L}\}KP\boldsymbol{A}\exp\{i\tau\boldsymbol{L}\}KA_t = \boldsymbol{T}_\tau A_t.$$

So the time-derivative of the quantity A_t is expressed only by A_t. The operator $\boldsymbol{T}$ which is generally nonlinear is still dependent on the time-interval $\tau = t - t_0$. For physical reasons assumptions about τ relative to the special system have to be made (e. g. $t_0 \to \infty$), which yield time-independent equations of motion, i. e. kinetic equations.

3. Classical Perturbation Theory

In this part the classical perturbation theory in functional formalism [6] will be treated in as much as it is necessary for the following investigations. The starting-point of the classical perturbation theory is the Liouville-equation (2.5). The complications in examining the temporal evolution of the statistical functional $F^{(N)}$ are caused by the interaction L^{v} in (2.5). Therefore it is useful to regard the undisturbed motion with $\exp(-i\tau\boldsymbol{L}^0)$ as zeroth order of a perturbation expansion relative to $\boldsymbol{L}^{\mathrm{v}}$. The $\boldsymbol{S}$-operator formalism known from quantum-theory yields with

$$\exp\{-i\tau\boldsymbol{L}\} \equiv \exp\{-i\tau\boldsymbol{L}^0\}\boldsymbol{S}_\tau,$$
$$i\frac{\partial}{\partial t}\boldsymbol{S}_\tau = \tilde{\boldsymbol{L}}^{\mathrm{v}}{}_\tau\boldsymbol{S}_\tau, \quad \boldsymbol{S}_0 = \boldsymbol{1},$$
$$\tilde{\boldsymbol{L}}^{\mathrm{v}}{}_\tau \equiv \exp\{i\tau\boldsymbol{L}^0\}\boldsymbol{L}^{\mathrm{v}}\exp\{-i\tau\boldsymbol{L}^0\} \tag{1}$$
$$= \int \frac{\mathrm{d}1\,\mathrm{d}2}{2!}\eta_1\eta_2\exp\{i\tau(\boldsymbol{l}_1+\boldsymbol{l}_2)\}\boldsymbol{l}_{12}\cdot\exp\{-i\tau(\boldsymbol{l}_1+\boldsymbol{l}_2)\}\frac{\delta}{\delta\eta_1}\frac{\delta}{\delta\eta_2}$$

the representation of the $\boldsymbol{S}$-operator

$$\boldsymbol{S}_\tau = \boldsymbol{1} + (-i)\int_0^\tau \mathrm{d}t_1\,\tilde{\boldsymbol{L}}^{\mathrm{v}}_{t_1} + (-i)^2\int_0^\tau \mathrm{d}t_1\,\tilde{\boldsymbol{L}}^{\mathrm{v}}_{t_1}\int_0^{t_1}\mathrm{d}t_2\,\tilde{\boldsymbol{L}}^{\mathrm{v}}_{t_2} + \ldots. \tag{2}$$

[14] E. T. JAYNES, Phys. Rev. **106**, 620 [1957]; **108**, 171 [1957].

[15] Similar consideration were made by R. M. LEWIS, J. Math. Phys. **8**, 1448 [1967]. We are indebted to Prof. L. WALDMANN for calling our attention to this paper.

With the aid of the general linked-cluster theorem [6,9] the series (2) can be summarized by

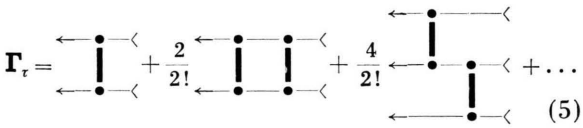
$$\mathbf{S}_\tau = N_{\eta,\,\delta/\delta\eta} \exp\{\mathbf{\Gamma}_\tau\}, \qquad \mathbf{\Gamma}_0 = 0\,. \tag{3}$$

Here $\mathbf{\Gamma}_\tau$ is the sum of all linked clusters of the series (2). The normal ordering operator $N_{\eta,\,\delta/\delta\eta}$ in (3) does not allow the functional derivatives appearing in $\mathbf{\Gamma}_\tau$ to be applied to $\mathbf{\Gamma}_\tau$ itself after expanding the series $\exp \mathbf{\Gamma}_\tau$. If one expresses $\widetilde{\boldsymbol{L}}^{\mathrm{v}}_\tau$ from (1) with the graphs from [6]

$$(-i)\int_0^\tau \mathrm{d}t_1\, \widetilde{\boldsymbol{L}}^{\mathrm{v}}_{t_1} = \tag{4}$$

on gets for $\mathbf{\Gamma}_\tau$ the series

$$\mathbf{\Gamma}_\tau = \tag{5}$$



where the sequence of interaction lines characterizes the sequence of the corresponding time-integrations [6]. With (1), (3), and (5) the functional $F^{(N)}_{t_0}$ is formally transformed into the functional $F^{(N)}_t$. By this, the factors $\exp\{\pm i\tau \boldsymbol{L}\}$ in (2.10) may be replaced by the expression $\exp\{\pm i\tau \boldsymbol{L}^\circ\}\,\mathbf{S}_{\pm\tau}$. Then the operators P and K remain to be investigated.

Since the quantity of interest here is the one-particle distribution f_1, the statistical functional resp. the N-particle distribution $f_{1\ldots N}$ has to be constructed, if only f_1 is known. This problem was solved in detail in [16]. The result in terms of information-theory is

$$f_{1\ldots N,\,t} = N!\,\frac{f_{1,t}}{N}\cdots\frac{f_{N,t}}{N}, \qquad F^{(N)}_t = \left[\int \mathrm{d}1\,\eta_1\,\frac{f_{1,t}}{N}\right]^N. \tag{6}$$

Finally, the reduced distribution functions may be obtained from $F^{(N)}_t$ according to [6] with

$$\begin{aligned} f_{1\ldots s,\,t} &= P\,\frac{\delta^s}{\delta\eta_1\ldots\delta\eta_s}\,F^{(N)}_t \\ &= P\,\frac{\delta^s}{\delta\eta_1\ldots\delta\eta_s}\exp\{-i\tau\,\boldsymbol{L}^\circ\}\,\mathbf{S}_\tau\,F^{(N)}_{t_0}, \end{aligned} \tag{7}$$

$$f_{1,\,t} = P\,\frac{\delta}{\delta\eta_1}\exp\{-i\tau\,\boldsymbol{L}^\circ\}\,\mathbf{S}_\tau\left[\int \mathrm{d}1'\,\eta_{1'}\,\frac{f_{1',\,t_0}}{N}\right]^N$$

resp. with

$$\begin{aligned} &f_{1\ldots s,\,t} \\ &\quad = \exp\{-i\tau(\boldsymbol{l}_1+\ldots+\boldsymbol{l}_s)\}\,P\,\frac{\delta^s}{\delta\eta_1\ldots\delta\eta_s}\,\mathbf{S}_\tau\,F^{(N)}_{t_0}, \end{aligned} \tag{8}$$

$$f_{1,\,t} = \exp\{-i\tau\,\boldsymbol{l}_1\}\,P\,\frac{\delta}{\delta\eta_1}\,\mathbf{S}_\tau\left[\int \mathrm{d}1'\,\eta_{1'}\,\frac{f_{1',\,t_0}}{N}\right]^N.$$

Now the necessary results of the classical perturbation theory are summarized. However, applying the $\mathbf{S}$-operator to the initial functional in (7) resp. (8) proves very complicated. Therefore an extension of this method to systems with an arbitrary number of particles shall be given. In both cases the correspondence is as good as with canonical and grand-canonical ensembles in equilibrium statistical mechanics.

4. Generalized Statistical Functional

The functional (2.4) may be generalized in reference to systems with an arbitrary number of particles by the definition

$$F_t = \sum_N^{0\ldots\infty} w^{(N)} \int \frac{\mathrm{d}1\ldots\mathrm{d}N}{N!}\,\eta_1\ldots\eta_N\,f_{1\ldots N,\,t}\,. \tag{1}$$

Here $f_{1\ldots N,\,t}$ means the distribution function in an N-particle phase-space, and $w^{(N)}$ means the probability to find just N particles in the system. Reduced distribution functions may be obtained from F_t with the same procedure as from the N-particle functional $F^{(N)}_t$. Since these generalized distribution functions have other properties than those following from $f_{1\ldots N}$ (vid. e. g. [17]), they are denoted by $n_{1\ldots s,\,t}$:

$$\begin{aligned} n_{1\ldots s,\,t} &= P\,\frac{\delta^s}{\delta\eta_1\ldots\delta\eta_s}\,F_t \\ &= \sum_N^{s\ldots\infty} w^{(N)} \int \frac{\mathrm{d}(s+1)\ldots\mathrm{d}N}{(N-s)\,!}\,f_{1\ldots N,\,t}\,. \end{aligned} \tag{2}$$

The operator P also sets all η_k equal to one after performing the functional derivatives. The generalized functional can be constructed with the generalized distribution functions $n_{1\ldots s}$ instead of $w^{(N)}$ and $f_{1\ldots N}$. For this purpose new functions ε are introduced

$$\eta \equiv 1+\varepsilon\,, \qquad \frac{\delta}{\delta\eta} = \frac{\delta}{\delta\varepsilon} \tag{3}$$

instead of the test-functions η. After evaluation, one gets

$$F_t = \sum_s^{0\ldots\infty} \int \frac{\mathrm{d}1\ldots\mathrm{d}s}{s!}\,\varepsilon_1\ldots\varepsilon_s\,n_{1\ldots s,\,t}\,. \tag{4}$$

Because of (3) the operator P sets all ε_i equal to zero. The $n_{1\ldots s}$ may also be obtained from (4) by the functional operators $\delta/\delta\varepsilon$.

[16] K. Voss, Ann. Phys. Leipzig **19**, 370 [1967].
[17] K. Voss, Wiss. Z. Techn. Univ. Dresden, to be published.

Knowledge of the one-particle distribution function n_1 at a certain time is not sufficient for an exact representation of the higher distribution functions and thus of the functional F_t. However, with the aid of information-theoretical methods [17] it can be shown that the $n_{1\ldots s}$ may be represented by the factorization-ansatz

$$n_{1\ldots s} = n_1 n_2 \ldots n_s , \tag{5}$$

if only n_1 is known. As in (3.6) the correlations are neglected in (5). With (4) and (5) the simple formula

$$F_t = \exp\{\int \mathrm{d}1\, \varepsilon_1 n_{1,t}\} \tag{6}$$

follows for F_t. The classical perturbation theory described in part 3 may be used directly for the generalized functional (1) or (6), because neither the Liouville-operator $\boldsymbol{L}$ nor the functional operator $\boldsymbol{S}$, nor the linked-cluster representation depend on the particle number N.

5. General Equation of Motion for the One-Particle Distribution

By time-differentiation of the evolution equation (3.8) especially for the generalized one-particle distribution $n_{1,t}$ the equation

$$i\frac{\partial}{\partial t} n_{1,t} = \boldsymbol{l}_1 n_{1,t} + \int \mathrm{d}2\, \boldsymbol{l}_{12} \exp\{-i\tau(\boldsymbol{l}_1+\boldsymbol{l}_2)\} \cdot P \frac{\delta^2}{\delta\eta_1 \delta\eta_2} \boldsymbol{S}_\tau F_{t_0} \tag{1}$$

which is known from the BBGKY-hierarchy is obtained by means of (3.1) and the properties of P. Because of $P\boldsymbol{S}_\tau = 1$ and $P\boldsymbol{\Gamma}_\tau = 0$, i. e. the properties of $\boldsymbol{S}_\tau$ and $\boldsymbol{\Gamma}_\tau$ following from the normalization, the relation

$$\begin{aligned} &P\frac{\delta}{\delta\eta_1}(N_{\eta,\delta/\delta\eta}\exp\{\boldsymbol{\Gamma}_\tau\})F_{t_0} = P[F_{t_0,1} + \boldsymbol{\Gamma}_{\tau,1}F_{t_0}] , \\ &P\frac{\delta^2}{\delta\eta_1\delta\eta_2}(N_{\eta,\delta/\delta\eta}\exp\{\boldsymbol{\Gamma}_\tau\})F_{t_0} \\ &\quad = P[F_{t_0,12} + \boldsymbol{\Gamma}_{\tau,1}F_{t_0,2} + \boldsymbol{\Gamma}_{\tau,2}F_{t_0,1} \\ &\quad\quad + (N_{\eta,\delta/\delta\eta}\boldsymbol{\Gamma}_{\tau,1}\boldsymbol{\Gamma}_{\tau,2})F_{t_0} + \boldsymbol{\Gamma}_{\tau,12}F_{t_0}] \end{aligned} \tag{2}$$

is obtained with the linked-cluster theorem. The indices of $F_{t_0,k}$ resp. $\boldsymbol{\Gamma}_{\tau,l}$ symbolize the corresponding functional derivatives $\delta F_{t_0}/\delta\eta_k$ resp. $\delta\boldsymbol{\Gamma}_\tau/\delta\eta_l$. Taking into account the form (4.6) of F_{t_0} one obtains with (4.3)

$$\begin{aligned} F_{t_0,1} &= n_{1,t_0}F_{t_0} , \\ F_{t_0,1} &= n_{1,t_0}n_{2,t_0}F_{t_0} . \end{aligned} \tag{3}$$

Therefore all operators $\delta/\delta\eta = \delta/\delta\varepsilon$ in the linked-clusters may be replaced by the corresponding one-particle functions. Finally, one has to put every η equal to one and every ε equal to zero because of P. With $\boldsymbol{\Gamma}_\tau$ $(\eta_k = 1,\ \delta/\delta\eta_l = n_{l,t_0}) \equiv \Gamma_\tau[n_{t_0}]$ from (2) one has obtained

$$\begin{aligned} P\frac{\delta}{\delta\eta_1}\boldsymbol{S}_\tau F_{t_0} &= n_{1,t_0} + \Gamma_{\tau,1}[n_{t_0}] , \\ P\frac{\delta^2}{\delta\eta_1\delta\eta_2}\boldsymbol{S}_\tau F_{t_0} &= (n_{1,t_0} + \Gamma_{\tau,1}[n_{t_0}]) \\ &\quad \cdot (n_{2,t_0} + \Gamma_{\tau,2}[n_{t_0}]) + \Gamma_{\tau,12}[n_{t_0}] . \end{aligned} \tag{4}$$

This last expression may be substituted easily into (1). However, simplifications are possible by using equations coresponding to (2.7) and (2.9)

$$\begin{aligned} n_{1,t} &= \exp\{-i\tau\boldsymbol{l}_1\}[n_{1,t_0} + \Gamma_{\tau,1}[n_{t_0}]] , \\ n_{1,t_0} &= \exp\{i\tau\boldsymbol{l}_1\}[n_{1,t} + \Gamma_{-\tau,1}[n_t]] . \end{aligned} \tag{5}$$

They were gained from (3.8) with the aid of (2), (3), and (4). The equation of motion (1) may be written with (4) and (5) in the form

$$\begin{aligned} i\frac{\partial}{\partial t} n_{1,t} &= \boldsymbol{l}_1 n_{1,t} + \int \mathrm{d}2\, \boldsymbol{l}_{12}[n_{1,t}n_{2,t} \\ &\quad + \exp\{-i\tau(\boldsymbol{l}_1+\boldsymbol{l}_2)\}\Gamma_{\tau,12}[n_{t_0}]] , \\ n_{1,t_0} &= \exp\{i\tau\boldsymbol{l}_1\}[n_{1,t} + \Gamma_{-\tau,1}[n_t]] . \end{aligned} \tag{6}$$

This equation contains a Vlasov-term. Because of the correlations and the mutual interaction of the particles the corrective term still depends on the one-particle distribution n_{1,t_0}, which, however, can be calculated from $n_{1,t}$.

Thus a general closed equation of motion (6) for the one-particle distribution $n_{1,t}$ can be obtained by using functional methods and by arguments of information-theory. The form of this equation is suitable for further investigations, because only the functional representation of $\Gamma_{\tau,12}[n_{t_0}]$ and $\Gamma_{-\tau,1}[n_t]$ has to be determined. An exact summation of all linked-clusters in (3.5) is impossible. However, partial summations may be performed, which at the same time afford the possibility to carry out the limit $\tau \to \infty$, in order to derive closed, explicitly time-independent equations, i. e. kinetic equations for the one-particle distribution. This program is contained in the following paper II.